Journal Title 1 (0) 1–4
IOS Press

DegreEmbed: incorporating entity embedding into logic rule learning for knowledge graph reasoning

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Abstract. Knowledge graphs (KGs), as structured representations of real world facts, are intelligent databases incorporating human knowledge that can help machine imitate the way of human problem solving. However, due to the nature of rapid iteration as well as incompleteness of data, KGs are usually huge and there are inevitably missing facts in KGs. Link prediction for knowledge graphs is the task aiming to complete missing facts by reasoning based on the existing knowledge. Two main streams of research are widely studied: one learns low-dimensional embeddings for entities and relations that can capture latent patterns, and the other gains good interpretability by mining logical rules. Unfortunately, previous studies rarely pay attention to heterogeneous KGs. In this paper, we propose DegreEmbed, a model that combines embedding-based learning and logic rule mining for inferring on KGs. Specifically, we study the problem of predicting missing links in heterogeneous KGs that involve entities and relations of various types from the perspective of the degrees of nodes. Experimentally, we demonstrate that our DegreEmbed model outperforms the state-of-the-art methods on real world datasets. Meanwhile, the rules mined by our model are of high quality and interpretability.

Keywords: Knowledge graph reasoning, Link prediction, Logic rule mining, Degree embedding, Interpretability

1. Introduction

Recent years have witnessed the growing attraction of knowledge graphs in a variety of applications, such as dialogue systems [1, 2], search engines [3] and domain-specific softwares [4, 5]. Capable of incorporating large-scale human knowledge, KGs provide graph-structured representation of data that can be comprehended and examined by human. Knowledge in KGs is stored in triple form (e_s, r, e_o), with e_s and e_o denoting subject and object entities and r a binary relation (*a.k.a.* predicate). For example, the fact that Mike is the nephew of Pete can be formed as $(\texttt{Mike}, \texttt{nephewOf}, \texttt{pete})$. However, due to the data explosion nowadays, it is impossible to collect all knowledge, causing incompleteness of KGs, that is, missing links in the graph, *e.g.*, the work of [6, 7] shows that there are more than 66% of the person entities missing a birthplace in two open KGs Freebase [8] and DBpedia [9].

Predicting missing triples based on the existing facts is usually called *link prediction* as a subtask of knowledge graph completion (KGC) [10], and numerous models have been developed for solving such problems. One prominent direction in this line of research is *representation learning* methods that learn distributed embedding vectors for entities and relations, such as TransE [11] and CompleX [12]. In this work, they are referred to as embedding-based methods. This kind of models are more capable of capturing latent knowledge through low-dimensional vectors, *e.g.* we can classify male and female entities in a family KG by clustering their points at the semantic space. In spite of achieving high performance, these

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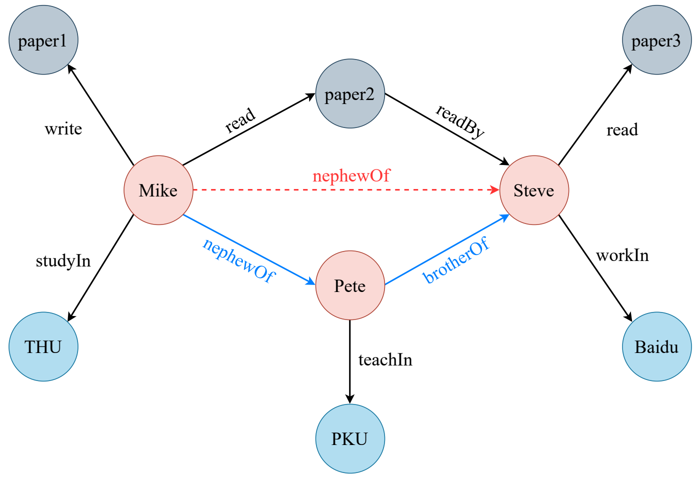

Fig. 1. An example of KG containing heterogeneous entities and relations: paper, person and institution. Entities in different colors mark their type. The existing links in the KG are presented as solid black lines, the missing one as dashed lines in red and the proper rule for inferring the link as blue lines.

models suffer from non-transparency and can poorly be understood by human, which is a common pain for most deep learning models.

Another popular approach is rule mining that discovers logical rules through mining co-occurrences of frequent patterns in KGs [13, 14]. This paper studies the problem of learning first-order logical Horn clauses for knowledge graph reasoning (KGR). As illustrated in Fig. 1, there is a missing link (*i.e.* `nephewOf`) between the subject `Mike` and the object `Steve`, but we can complete the fact through a logic rule `nephewOf(Mike, Pete)` ∧ `brotherOf(Pete, Steve)` ⇒ `nephewOf(Mike, Steve)`, meaning that if Mike is the nephew of Pete and Steve has a brother Pete, then we can infer that Mike is the nephew of Steve. Reasoning on KGs through Horn clauses has been previously studied in the area of *Inductive Logic Programming* [15]. One representative method, Neural LP [16], is the first fully differentiable neural system that successfully combines learning discrete rule structures as well as confidence scores in continuous space. Although learning logical rules equips models with strong interpretability and the ability to generalize to similar tasks [17, 18], these methods often focus only on the relations of which the rules are made up, while the properties of the involved entities are not considered. For example, in the KG shown in Fig. 1, it is definitely wrong to infer by a rule containing a female-type relation path like `sisterOf` starting from Mike, because Mike is the nephew of Pete, which indirectly tells us he is a male. This sort of deficiency is more severe in heterogeneous KGs where there are entities and relations of different types mixing up. In these KGs, there might be multiple rules of no use, which decreases the performance and interpretability of ILP models.

In this paper, in order to bridge the gap between the two lines of research mentioned above, we propose DegreEmbed, a model of logic rule learning that integrates the inner attributes of entities by embedding nodes in the graph from the prospective of their degrees. DegreEmbed is not only interpretable to human, but also able to mine potential attributes of entities. We also evaluate our model on several knowledge graph datasets, and show that we are able to learn more accurately, and meanwhile, gain strong interpretability of mined rules.

2. Related work

Relational data mining. The problem of learning relational rules has been traditionally addressed in the field of *inductive logic programming* (ILP) [15]. These methods often view the task of completing a missing triple as a query $q(h,t)$ where they learn a probability as a confidence score for each rule between query entities and answer entities. Among these studies, Path-Ranking Algorithm (PRA) [19] investigated the framework of random walk inference, where they select a relational path under a set of constraints and perform maximum-likelihood classification. A RNN model was developed by [20] to compose the semantics of relations for arbitrary-length reasoning. Chain-

Reasoning proposed by [21], enabling multi-hop reasoning through a neural attention mechanism, reveals logical rules across all relations and entities. Although ILP models are capable of mining interpretable rules for human, these models typically take both positive and negative examples for training and suffer from a potentially large version space, which is a critical shortage since most modern KGs are huge and contain only positive instances.

Neural logic programming. Extending the idea of TensorLog that tackles the problem of rule-based logic reasoning through sparse matrix multiplication, Neural LP [16] is the first end-to-end differentiable approach to simultaneously learn continuous parameters and discrete structure of rules. Some recent methods [22–24] have improved the framework done by Neural LP [16] in different manners. DRUM [22] introduces tensor approximation for optimization and reformulate Neural LP to support rules of varied lengths. Neural-Num-LP [23] extends Neural LP to learn numerical relations like *age* and *weight* with dynamic programming and cumulative sum operation. NLIL [24] proposes a multi-hop reasoning framework for general ILP problem through a divide-and-conquer strategy as well as decomposing the search space into three subspaces. However, the existing methods ignore the effects caused by the entities while reasoning over a specific relational path, thus witness a more obvious failure where heterogeneous entities and relations are involved in the KGs.

Representation learning. Capturing their semantic information by learning low-dimensional embeddings of entities and relations, also known as *knowledge graph embedding*, is a vital research issue in KGR, and we term those models as embedding-based models. Newly proposed methods, including RotatE [25], ConvE [26] and TuckER [27], predict missing links by learning embedding vectors from various perspectives of the problem. Specifically, the work of RotatE [25] focuses on inferring patterns such as symmetry and inversion, where they proposed a rotational model that rotates the relation from the subject to the object in the complex space as $e_o = e_s \circ r$ where the $\circ$ denotes the element-wise Hadmard product. ConvE introduces a highly parameter efficient model, which uses 2D convolution over embeddings and multiple layers of nonlinear features to express semantic information. TuckER, inspired by the Tucker decomposition [28] that factorizes a tensor into a core tensor along with a set of matrices, is a linear model for link prediction that has good expressive power. Unfortunately, the biggest problem is that these sort of methods can hardly be comprehended by human, but we relate to these methods for their ability to capture latent information of entities and relations through embedding.

3. Preliminaries

3.1. Knowledge graph reasoning

Knowledge graph can be modeled as a collection of factual triples $\mathcal{G} = \{(e_s, r, e_o) \mid e_s, e_o \in \mathcal{E}, r \in \mathcal{R}\}$, with $\mathcal{E}, \mathcal{P}$ representing the set of entities and binary relations (*a.k.a.* predicates) respectively in the knowledge graph, and (e_s, r, e_o) the triple $(\texttt{subject}, \texttt{relation}, \texttt{object})$ in form of $e_s \xrightarrow{r} e_o$. During reasoning over KGs, each triple is usually presented in the form $\texttt{r}(e_s, e_o)$. The subgraph regarding a specific relation $\texttt{r}_i$ is described as a subset of $\mathcal{G}$ containing all triples with $\texttt{r}_i$ being the connection between the subject and object: $\mathcal{G}(\texttt{r}_i) = \{(e_s, r, e_o) \mid e_s, e_o \in \mathcal{E}, \texttt{r}_i \in \mathcal{R}, r = \texttt{r}_i\}$.

Logic rule reasoning. We perform reasoning on KGs by learning a *confidence* score $\alpha \in [0, 1]$ for a first-order logic *rule* in the form

$$\texttt{r}_1(x, z_1) \wedge \cdots \wedge \texttt{r}_l(z_{l-1}, y) \Rightarrow q(x, y) \ : \ \alpha, \quad (1)$$

$\mathbf{r(x, y)} \Rightarrow q(x, y)$ for short, with $r_1, \ldots, r_l, q \in \mathcal{R}$, $z_1, \ldots, z_{l-1} \in \mathcal{E}$, where $\mathbf{r} = \wedge_i r_i$, is called a `rule pattern`. For example, the rule $\texttt{brotherOf}(x, z) \wedge \texttt{fatherOf}(z, y) \Rightarrow \texttt{uncleOf}(x, y)$ intuitively states that if x is the brother of z and z is the father of y, then we can conclude that x is the uncle of y. All rule patterns of length l ($l \geqslant 2$) can be formally defined as a set of relational tuples $\mathcal{H}^l = \{(r_1, r_2, \ldots, r_l) \mid r_i \in \mathcal{R}, 1 \leqslant i \leqslant l\} = \mathcal{R}^l$, and the set of patterns no longer than L is denoted as $\mathbb{H}^L = \bigcup_{l=2}^{L} \mathcal{H}^l$. A `rule path` $\mathbf{p}$ is an instance of a certain pattern $\mathbf{r}$ via different sequences of entities, which can be denoted as $\mathbf{p} \triangleright \mathbf{r}$, *e.g.*, $(r_a(x, z_1), r_b(z_1, y))$ and $(r_a(x, z_2), r_b(z_2, y))$ are different paths of the same pattern.

The knowledge graph reasoning task here is considered to contain a variety of queries, each of which is composed of a `query` body $q \in \mathcal{R}$, an entity `head` h which the query is about, and an entity `tail` t that is the answer to the query such that $(h, q, t) \in \mathcal{G}$. Finally we want to find the most possible logic rules $h \xrightarrow{r_1} \cdots \xrightarrow{r_l} t$ to predict the link q. Thus, given maximum length L, we assign a single *confidence* score (*i.e.* probability) to a set of rule paths $\mathbf{p}$'s adhering to the same pattern $\mathbf{r}$ that connects h and t:

$$\{\mathbf{p}_i(h,t) \Rightarrow q(h,t) \mid \mathbf{p}_i \rhd \mathbf{r}, \mathbf{r} \in \mathbb{H}^L\} \ : \ \alpha \qquad (2)$$

During inference, given an entity h, the unified score of the answer t can be computed by adding up the confidence scores of all rule paths that infer $q(h,t)$, and the model will produce a ranked list of entities where higher the score implies higher the ranking.

3.2. Graph structure

Definition 1 (Directed Labeled Multigraph). *A **directed labeled multigraph** G is a tuple $G = (V, E)$, where V denotes the set of vertices, and $E \subseteq V \times V$ is a multiset of directed, labeled vertex pairs (i.e. edges) in the graph G.*

Because of its graph structure, a knowledge graph can be regarded as a directed labeled multigraph [29]. In this paper, "graph" is used to refer to "directed labeled multigraph" for the sake of simplicity. $G(\mathtt{r}) = (V(\mathtt{r}), E(\mathtt{r}))$ is the corresponding graph structure of $\mathcal{G}(\mathtt{r})$. $m = |V|$ and $n = |E|$ stand for the **number of vertices** and **number of edges** respectively for a graph G. Particularly in a KG, $|\mathcal{E}| = m$ and the total number of triplets (e_s, r, e_o) equals the number of edges, *i.e.* $|\mathcal{G}| = n$.

Formally, in a graph $G = (V, E)$, the **degree** of a vertex $v \in V$ is the number of edges incident to it. When it comes to directed graphs, **in-degree** and **out-degree** of a vertex v is usually distinguished, which are defined as

$$deg^{+}(v) = |\{(u,v) \mid \exists u \in V, (u,v) \in E\}| \qquad (3)$$

$$deg^{-}(v) = |\{(v,u) \mid \exists u \in V, (v,u) \in E\}| \qquad (4)$$

But in this paper, we use "degree" to represent the edges incident to a specific node v for conciseness.

3.3. Saturation

In the following, we will introduce a set of indicators to help evaluate the interpretability of a logic rule learning model. More specifically, these computational methods analyse the reasoning complexity from the inherent attributes of the graph structure G corresponding to a KG $\mathcal{G}$.

Definition 2 (Macro Reasoning Saturation). *Given a query $q \in \mathcal{R}$ and the maximum length L of a rule pattern $\mathbf{r}_l \in \mathbb{H}^L$, the **macro reasoning saturation** of $\mathbf{r}_l$ in relation to predicate q, i.e. $\gamma_q^{\mathbf{r}_l}$, is the percentage of triples (h_i, q, t_j) in subgraph $\mathcal{G}(q)$ such that $\mathbf{r}_l(h_i, t_j) \Rightarrow q(h_i, t_j)$.*

We can compute the macro reasoning saturation $\gamma_q^{\mathbf{r}_l}$ using the following equation:

$$\gamma_q^{\mathbf{r}_l} = \frac{|U^{\mathbf{r}_l}|}{n^q}, \qquad (5)$$

with $U^{\mathbf{r}_l}$ being the set $U^{\mathbf{r}_l} = \{(h,q,t) \mid (h,q,t) \in \mathcal{G}(q), \mathbf{r}_l(h,t) \Rightarrow q(h,t)\}$ that collects the factual triplets in $\mathcal{G}(q)$ as the reasoning candidates of rule $\mathbf{r}_l$, and $n^q = |\mathcal{G}(q)|$ being the number of edges (*i.e.* the number of triples) in $G(q)$. We can reasonably say that the larger $\gamma_q^{\mathbf{r}_l}$ grows, the more likely $\mathbf{r}_l$ can be as a proper inference of the query q. When $\gamma_q^{\mathbf{r}_l}$ equals 1, it means we can reason out every factual triples in $\mathcal{G}(q)$ through at least one rule path following the pattern $\mathbf{r}_l$.

Definition 3 (Micro Reasoning Saturation). *Given the maximum length L of a rule pattern, we define the micro reasoning saturation of pattern $\mathbf{r}_l \in \mathbb{H}^L$ as following. Firstly, for a specific triple $\mathtt{tri} = (h,q,t) \in \mathcal{G}$, i.e. $\delta_{\mathtt{tri}}^{\mathbf{r}_l}$, is the percentage of the paths $\mathbf{p}_{l_i} \rhd \mathbf{r}_l$ such that $\mathbf{r}_l(h,t) \Rightarrow q(h,t)$ as to all paths from h to t.*

The equation to compute $\delta_{\mathtt{tri}}^{\mathbf{r}_l}$ is

$$\delta_{\mathtt{tri}}^{\mathbf{r}_l} = \frac{|V^{\mathbf{r}_l}|}{|V^L|} \qquad (6)$$

where $V^{\mathbf{r}_l} = \{\mathbf{p}_{l_i} \mid \mathbf{p}_{l_i} \rhd \mathbf{r}_l, \mathbf{r}_l(h,t) \Rightarrow q(h,t)\}$, $V^L = \{\mathbf{p}_{k_j} \mid \mathbf{p}_{k_j} \rhd \mathbf{r}_k, \forall \mathbf{r}_k \in \mathbb{H}^L, \mathbf{r}_k(h,t) \Rightarrow q(h,t)\}$. $V^{\mathbf{r}_l}$ denotes the set of rule paths derived from the pattern $\mathbf{r}_l$ that is able to infer the fact (h,q,t), and V^L involve all the rules with their lengths no longer that L.

Then, we average $\delta_{\mathtt{tri}}^{\mathbf{p}_r}$ on all triples $(h,q,t) \in \mathcal{G}(q)$ and get the **micro reasoning saturation** of the pattern $\mathbf{r}_l \in \mathbb{H}^L$ for query q:

$$\delta_q^{\mathbf{r}_l} = \frac{1}{n^q} \sum_{\mathtt{tri} \in \mathcal{G}(q)} \delta_{\mathtt{tri}}^{\mathbf{r}_l} \qquad (7)$$

In Eqs. (5) and (7), $\gamma_q^{\mathbf{r}_l}$ and $\delta_q^{\mathbf{r}_l}$ assess how easy it is to infer q following the pattern $\mathbf{r}_l$ respectively from a macro and a micro perspective. The higher the two indicators are, the easier for models to gain the inference $\mathbf{r}_l(h,t) \Rightarrow q(h,t)$. In order to obtain an overall result,

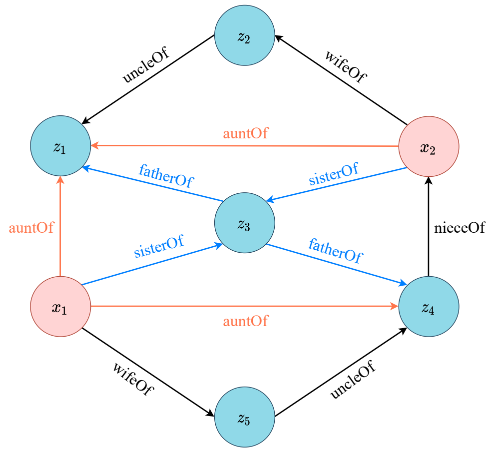

Fig. 2. A KG example of family members and the relations between them.

we define the comprehensive reasoning saturation $\eta_q^{\mathbf{r}_l}$ by combining the two indicators through multiplication.

$$\eta_q^{\mathbf{r}_l} = \gamma_q^{\mathbf{r}_l} \times \delta_q^{\mathbf{r}_l} \tag{8}$$

We can take the relation $q = \texttt{auntOf}$ and the rule $\mathbf{r}_l = \texttt{sisterOf} \wedge \texttt{fatherOf}$ in Fig. 2 as an example to show the computation of saturations. In subgraph $\mathcal{G}(q)$, there are totally three triples (presented in red), thus $n^q = 3$. For the triple $(x_2, \texttt{auntOf}, z_2)$, two rule paths can contribute to its inference: $\texttt{wifeOf}(x_2, z_1) \wedge \texttt{uncleOf}(z_2, z_1)$ and $\texttt{sisterOf}(x_2, z_3) \wedge \texttt{fatherOf}(z_3, z_1)$. In the same way, we can see there are one and two rule paths for $(x_1, \texttt{auntOf}, z_1)$ and $(x_1, \texttt{auntOf}, z_4)$ respectively. The rule $\mathbf{r}_l = \texttt{sisterOf} \wedge \texttt{fatherOf}$ appears as an inference among all these three triples, therefore the macro saturation is $\gamma_q^{\mathbf{r}_l} = 3/n^q = 100\%$. More detailed information can be extracted through computing the micro saturation. The rule $\mathbf{r}_l$ takes a percentage of 50% among all paths for the triple $(x_2, \texttt{auntOf}, z_1)$, while 100% and 50% for the other two triples. Thus, the micro saturation of $\mathbf{r}_l$ for q is $\delta_q^{\mathbf{r}_l} = (0.5 + 1 + 0.5)/n^q = 67\%$. Finally, we can compute the comprehensive saturation $\eta_q^{\mathbf{r}_l} = \gamma_q^{\mathbf{r}_l} \times \delta_q^{\mathbf{r}_l} = 67\%$.

4. A novel model for multi-target learning of logical rules for KGR

4.1. Neural LP for logic reasoning

4.1.1. TensorLog

The work of TensorLog [30, 31] successfully simulates the reasoning process using first-order logic rules by performing sparse matrix multiplication, based on which, Neural LP [16] proposed a fully differentiable reasoning system. In the following, we will first introduce the TensorLog operator. In a KG involving a set of entities $\mathcal{E}$ and a set of relations $\mathcal{R}$, factual triplets with respect to the relation r_k are restored in a binary matrix $\mathrm{M}_{r_k} \in \{0,1\}^{|\mathcal{E}|\times|\mathcal{E}|}$. M_{r_k}, an adjacency matrix, is called a TensorLog operator meaning that (e_i, r_k, e_j) is in the KG if and only if the (i, j)-th entry of M_{r_k} is 1. Let $\mathrm{v}_{e_i} \in \{0,1\}^{|\mathcal{E}|}$ be the one-hot encoded vector of entity e_i, then $s^\top = \mathrm{v}_{e_i}^\top \mathrm{M}_{r_1}\mathrm{M}_{r_2}\mathrm{M}_{r_3}$ is the *path features vector* [24], where the j-th entry counts the number of unique paths following the pattern $r_1 \wedge r_2 \wedge r_3$ from e_i to e_j [32].

For example, every KG entity $e \in \mathcal{E}$ in Fig. 2 is encoded into a 0-1 vector of length $|\mathcal{E}| = 7$. For every relation $r \in \mathcal{R}$ and every pair of entities $e_i, e_j \in \mathcal{E}$, the TensorLog operator relevant to r is define as a sparse matrix M_r with its (i, j)-th element being 1 if $(e_i, r, e_j) \in \mathcal{G}$. Considering the KG in Fig. 2, for the relation $r = \texttt{auntOf}$ we have

$$\mathrm{M}_r = \begin{array}{c} \\ \end{array}\begin{array}{ccccccc} x_1 & x_2 & z_1 & z_2 & z_3 & z_4 & z_5 \end{array}$$
$$\mathrm{M}_r = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} x_1 \\ x_2 \\ z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{matrix}$$

And the rule $\texttt{sisterOf}(\mathrm{X}, \mathrm{Z}) \wedge \texttt{fatherOf}(\mathrm{Z}, \mathrm{Y}) \Rightarrow \texttt{auntOf}(\mathrm{X}, \mathrm{Y})$ can be simulated by performing the following sparse matrix multiplication:

$$\mathrm{M}_{\mathrm{r}'} = \mathrm{M}_{\mathrm{sisterOf}}\,\mathrm{M}_{\mathrm{fatherOf}} =$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} x_1 \\ x_2 \\ z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{matrix}$$
$$\begin{matrix} x_1 & x_2 & z_1 & z_2 & z_3 & z_4 & z_5 \end{matrix}$$

By setting $\mathrm{v}_{x_1} = [1,0,0,0,0,0,0]^\top$ as the one-hot vector of x_1 and multiplying by $\mathrm{v}_{x_1}^\top$ on the left, we obtain $s^\top = \mathrm{v}_{x_1}^\top \cdot \mathrm{M}_{\mathrm{r}'} = [0,0,1,0,0,1,0]$. The resultant $s^\top$ selects the row is actually $\mathrm{M}_{\mathrm{r}'}$ identified by x_1. By operating right-hand side multiplication with v_{z_1}, we get the number of unique paths following the pattern `sisterOf` $\wedge$ `fatherOf` from x_1 to z_1: $s^\top \cdot \mathrm{v}_{z_1} = 1$.

4.1.2. Neural LP

Neural LP [16] inherits the idea of TensorLog. Given a query $q(h,t)$, after L steps of reasoning, the score of the query induced through rule pattern $\mathbf{r}_s$ of length L is computed as

$$\mathrm{score}(t \mid q, h, \mathbf{r}_s) = \mathrm{v}_h^\top \prod_{l=1}^{L} \mathrm{M}^l \cdot \mathrm{v}_t, \tag{9}$$

where M^l is the adjacency matrix of the relation used at l-th hop.

The operators above are used to learn for query q by calculating the weighted sum of all possible patterns:

$$\sum_s \alpha_s \prod_{k\in\beta_s} \mathrm{M}_{r_k}, \tag{10}$$

where s indexes over all potential patterns with maximum length of L, α_s is the confidence score associated with the rule $\mathbf{r}_s$ and β_s is the ordered list of relations appearing in $\mathbf{r}_s$.

To summarize, we update the score function in Eq. (9) by finding an appropriate α in

$$\varphi(t \mid q, h) = \mathrm{v}_h^\top \sum_s \alpha_s \cdot \left(\prod_{k\in\beta_s} \mathrm{M}_{r_k} \cdot \mathrm{v}_t \right), \tag{11}$$

and the optimization objective is

$$\max_{\alpha_s} \sum_{(h,q,t)\in\mathcal{G}} \varphi(t \mid q, h), \tag{12}$$

where α_s is to be learned.

Whereas the searching space of learnable parameters is exponentially large, *i.e.* $O(|\mathcal{R}|^L)$, direct optimization of Eq. (12) may fall in the dilemma of over-parameterization. Besides, it is difficult to apply gradient-based optimization. This is because each variable α_s is bound with a specific rule pattern, and it is obviously a discrete work to enumerate rules. To overcome these defects, the parameter of rule $\mathbf{r}_s$ can be reformulated by distributing the confidence to its inclusive relations at each hop, resulting in a differentiable score function:

$$\phi_L(t \mid q, h) = \left(\mathrm{v}_h^\top \prod_{l=1}^{L} \sum_{k=0}^{|\mathcal{R}|} a_k^l \mathrm{M}_{r_k} \right) \cdot \mathrm{v}_t, \tag{13}$$

where L is a hyperparameter denoting the maximum length of patterns and $|\mathcal{R}|$ is the number of relations in KG. M_{r_0} is an identity matrix I that enables the model to include all possible rule patterns of length L or smaller [22].

To perform training and prediction over the Neural LP framework, we should first construct a KG from a large subset of all triplets. Then we remove the edge (h,t) from the graph when facing the query (h,q,t), so that the score of t can get rid of the influence imposed from the head entity h directly through the edge (h,t) for the correctness of reasoning.

4.2. DegreEmbed model

In this section, we propose our *DegreEmbed* model based on Neural LP [16] as a combination of embedding-based models and ILP models where the potential properties of individual entities are considered through entity degree embedding. We discover that the attributes of nodes in a KG can make a difference via observation on their degrees. In Fig. 1, we notice that Mike is a male because he is a nephew of someone, hence it is incorrect indeed to reason by a rule containing a female-type relation starting from Mike. Also, the in-degree (*i.e.* `studyIn`) of entity THU proves its identity as a university. Besides, as illustrated in Section 4.1.1, the inherent properties of a KG are stored in the relational matrices, which is our aim to reconstruct for harboring type information of entities. For a query $q(h,t)$, the final score is a scalar obtained by Eq. (13), where the *path feature vector* is $s^\top = \mathrm{v}_h^\top \prod_{l=1}^{L} \sum_{k=0}^{|\mathcal{R}|} a_k^l \mathrm{M}_{r_k}$, and v_t selects the t-th scalar of $s^\top$ through matrix multiplication. In fact, the vector $s^\top \in \mathbb{R}^{|\mathcal{E}|}$ is a row of matrix $\prod_{l=1}^{L} \sum_{k=0}^{|\mathcal{R}|} a_k^l \mathrm{M}_{r_k}$, each value of which is the "influence" passed from the head entity h to the certain entity. As a result, we can add attributes of the entity e_i by changing the i-th row of adjacency matrices from the prospective of the type of degrees of e_i.

For any entity $e \in \mathcal{E}$, we collect the ones of unique types among all of its in- and out-degrees separately. Suppose the number of unique degrees is d, then we project them onto a semantic space by looking up in a row-vector embedding matrix $\mathbf{E}^{|\mathcal{R}|\times m}$, and the result is number of d vectors arranged in a matrix $\mathbf{M} \in \mathbb{R}^{d\times m}$, where m is the embedding dimension. The embedding vectors are input into a bidirectional LSTM [33] at different time steps. Finally, we perform the attention operation on the hidden state of the BiLSTM at the last time step to get the $|\mathcal{R}|$-dimensional *degree feature vector* of e for $1 \leqslant i \leqslant d$:

$$\mathbf{h}_i, \mathbf{h}'_{d-i+1} = \mathrm{BiLSTM}(\mathbf{h}_{i-1}, \mathbf{h}'_{d-i}, \mathbf{M}), \qquad (14)$$

$$\rho_e = \mathrm{softmax}(WH + b), \qquad (15)$$

where $\mathbf{h}$ and $\mathbf{h}'$ are the hidden states of the forward and backward path LSTMs, with the subscripts denoting their time step, and H is obtained by concatenating $\mathbf{h}_d$ and $\mathbf{h}'_1$. $\rho_e \in \mathbb{R}^{|\mathcal{R}|}$ is the *degree feature vector* of entity e, whose elements can be viewed as the weights for relations. At last, we replace the elements that are in the row identified by e and equal 1 in each adjacency matrix M_{r_k} by the k-th value of ρ_e. By following the same procedure for the other entities in the KG, we construct a new set of relational matrices $B_{r_1}, \dots, B_{r_{|\mathcal{R}|}}$, which are called *DegreEmbed* operators. The score function shown in Eq. (13) is updated accordingly as follows:

$$\phi'_L(t \mid q, h) = \left(\mathrm{v}_h^\top \prod_{l=1}^{L} \sum_{k=0}^{|\mathcal{R}|} a_k^l \mathrm{B}_{r_k} \right) \cdot \mathrm{v}_t, \qquad (16)$$

where the $\mathrm{B}_{r_1}, \dots, \mathrm{B}_{r_{\mathcal{R}}}$ is our new DegreEmbed operators, and B_{r_0} is still the identity matrix. The whole process to compute the operators makes it possible to incorporate the information of entities for rule learning models, where the degree feature vector ρ_e can be viewed as the embedding vector of the entity e. Remarkably, the DegreEmbed operators can be pretrained due to its belonging to the inner attribute of a KG, thus resulting in a model that can be easily transferred to other tasks. An overview of computing the DegreEmbed operators is illustrated in Fig. 3

Finally, the confidence scores are learned over the bidirectional LSTM followed by the attention using Eqs (17) and (18) for the temporal dependency among several consecutive steps. The `input` in Eq. (17) is `query` embedding from another lookup table. For $1 \leqslant i \leqslant L$ we have

$$\mathbf{h}_i, \mathbf{h}'_{L-i+1} = \mathrm{BiLSTM}(\mathbf{h}_{i-1}, \mathbf{h}'_{L-i}, \mathrm{input}), \qquad (17)$$

$$[a_{i,1}, \dots, a_{i,|\mathcal{R}|}] = f_\theta\left([\mathbf{h}_i \parallel \mathbf{h}'_{L-i}]\right), \qquad (18)$$

where $[a_{i,1}, \dots, a_{i,|\mathcal{R}|}]$ is the attention vector obtained by performing a linear transformation over concatenated forward and backward hidden states, followed by a softmax operator: $f_\theta(H) = \mathrm{softmax}(WH + b)$.

4.3. Optimization

Loss construction. In general, this task of KGC is treated as a classification of entities to build the loss. For each query $q(h,t)$ in KG, we first split the objective function Eq. (16) into two parts: target vector v_t and prediction vector

$$s^\top = \mathrm{v}_h^\top \prod_{l=1}^{L} \sum_{k=0}^{|\mathcal{R}|} a_k^l \mathrm{B}_{r_k}, \qquad (19)$$

and then our goal is to minimize the cross-entropy loss between v_t and $\mathrm{s}^\top$:

$$\ell_q(h,t) = -\sum_{i=1}^{|\mathcal{E}|} \{\mathrm{v}_t[i] \cdot \log(\mathrm{s}[i]) + (1 - \mathrm{v}_t[i]) \cdot \log(1 - \mathrm{s}[i])\},$$

where i indexes elements in vector v_t and s.

Low-rank approximation. It can be shown that the final confidences obtained by expanding ϕ'_L are a rank one estimation of the *confidence value tensor* [22], and a low-rank approximation is a popular method for tensor approximation. Hence we follow the work of [22] and rewrite Eq. (16) using rank T approximation, as shown in Eq. (20).

$$\Phi_L(t \mid q, h) = \left(\mathrm{v}_h^\top \sum_{j=1}^{T} \prod_{l=1}^{L} \sum_{k=0}^{|\mathcal{R}|} a_{j,k}^l \mathrm{B}_{r_k} \right) \cdot \mathrm{v}_t, \qquad (20)$$

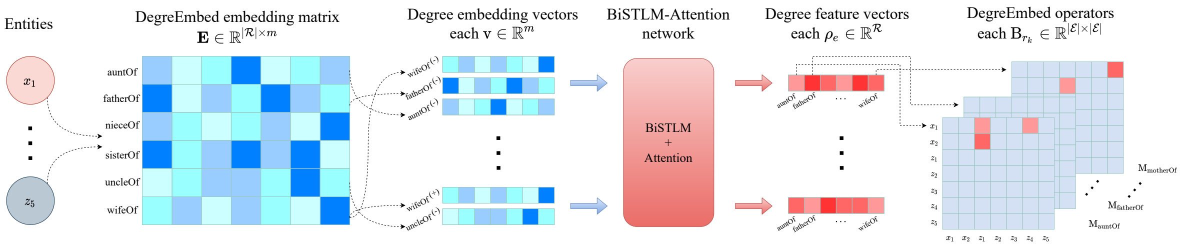

Fig. 3. An illustration of computing the DegreeEmbed operators for the KG shown in Fig. 2. Superscripts (+) and (-) of the labels of degree embedding vectors denote their in and out direction. All DegreeEmbed operators are initialized to zero matrices.

More concretely, we update Eqs. (17) and (18), as is shown in Eqs. (21) and (22), by deploying number of T BiLSTMs of the same network structure, each of which can extract features from various dimensions.

$$\mathbf{h}_i^{(j)}, \mathbf{h}_{L-i+1}^{'(j)} = \mathrm{BiLSTM}_j(\mathbf{h}_{i-1}^{(j)}, \mathbf{h}_{L-i}^{'(j)}, \mathrm{input}) \quad (21)$$

$$[a_{i,1}^{(j)}, \ldots, a_{i,|\mathcal{R}|}^{(j)}] = f_\theta\left([\mathbf{h}_i^{'(j)} \mid\mid \mathbf{h}_{L-i}^{'(j)}]\right), \quad (22)$$

where the superscripts of the hidden states identify their bidirectional LSTM.

5. Experiment

In this section, we report the evaluation results of our model on a knowledge graph completion task, where we compare the effectiveness of our model against the state-of-the-art KGR learning systems. Meanwhile, as DegreEmbed takes advantage in the interpretability in contrast to embedding-based methods, we also examine the rules mined by DegreEmbed with the help of the indicator *saturation*, which assesses the quality of rules from the graph structure of a KG.

5.1. Experiment setting

5.1.1. Knowledge graph completion

The knowledge graph completion task we use is a canonical one as described in [16]. When training the model, the `query` and `head` are part of incomplete triplets for training, and the goal is to find the most possible entity as the answer `tail`. For example, if `nephewOf(Mike, Steve)` is missing from the knowledge graph, our goal is to learn rules for reasoning over the existing KG and retrieve `Steve` when presented with the query `nephewOf(Steve, ?)`. For each triplet (h, q, t), two queries are designed as $(h, q, ?)$ and $(?, q, t)$ with answers t and h for data augmentation. During evaluation, for each query, we manually remove the edge (h, t) from KG for the correctness of reasoning results and the score is computed for each entity, as well as the rank of the correct answer. For the computed ranks from all queries, we report the Mean Reciprocal Rank (MRR) and Hit@k under the filtered protocol [11]. MRR averages the reciprocal ranks of the answer entities and Hit@k computes the percentage of how many desired entities are ranked among top k.

5.1.2. Datasets

To evaluate our method for learning logic rules in heterogeneous KGs, we select the following datasets for knowledge graph completion task:

- *FB15K-237* [34], a more challenging version of FB15K [11] based on Freebase [8], a growing knowledge graph of general facts.
- *WN18* [26], a subset of knowledge graph WordNet [35, 36] constructed for a widely used dictionary.
- *Medical Language System (UMLS)* [37], from biomedicine, where the entities are biomedical concepts (*e.g.* `organism`, `virus`) and the relations consist of `affects` and `analyzes`, etc.
- *Kinship* [37], containing kinship relationships among members of a Central Australian native tribe.
- *Family* [37], containing individuals from multiple families that are biologically related.

Statistics about each dataset used in our experiments are presented in Table 1. All datasets are randomly split into 4 files: *facts*, *train*, *valid* and *test*, and the ratio is 6:2:1:1. The *facts* file contains a relatively large proportion of the triplets for constructing the KG. The *train* file is composed of query examples $q(h, t)$. The *valid* and *test* files both contain queries $q(h, t)$, in which the former is used for early stopping and the lat-

Table 1
Statistics of datasets.

Dataset	# Relation	# Entity	# Triplets	# Facts	# Train	# Validation	# Test
FB15K-237	237	14541	310116	204087	68028	17535	20466
WN18	18	40943	151442	106088	35353	5000	5000
Family	12	3007	28356	17615	5868	2038	2835
Kinship	25	104	10686	6375	2112	1099	1100
UMLS	46	135	6529	4006	3009	569	633

Table 2
Links to the models used in this work.

Model	Link
TransE, DistMult and ComplEx	https://github.com/Accenture/AmpliGraph
TuckER	https://github.com/ibalazevic/TuckER
RotatE	https://github.com/liyirui-git/KnowledgeGraphEmbedding_RotatE
RNNLogic	https://github.com/DeepGraphLearning/RNNLogic
Neural LP	https://github.com/fanyangxyz/Neural-LP
DRUM	https://github.com/alisadeghian/DRUM

ter is for testing. Unlike the case of learning embeddings, our method does not necessarily require the entities in *train*, *valid* and *test* to overlap.

5.1.3. Comparison of algorithms

In experiment, the performance of our model is compared with that of the following algorithms:

- Rule learning algorithms. Since our model is based on neural logic programming, we choose Neural LP and a Neural LP-based method DRUM [22]. We also consider a probabilistic model called RNNLogic [38].
- Embedding-based algorithms. We choose several embedding-based algorithms for comparison of the expressive power of our model, including TransE [11], DistMult [39], ComplEx [12], RotatE [25] and TuckER [27].

The implementations of the above models we use are available at the links listed in App. 2.

5.1.4. Model configuration

Our model is implemented using PyTorch [40] and the code will be publicly available. We use the same hyperparameter setting during evaluation on all datasets. The query and entity embedding have the dimension 128 and are both randomly initialized. The hidden state dimension of BiLSTM(s) for entity and degree embedding are also 128. As for optimization algorithm, we use mini-batch ADAM [41] with the batch size 128 and the learning rate initially set to 0.001. We also observe that the whole model tends to be more trainable if we use the normalization skill.

Note that Neural LP [16] and DRUM [22] and our method all conform to a similar reasoning framework. Hence, to reach a fair comparison, we ensure the same hyperparameter configuration during experiments on these models, where the maximum rule length L is 2 and the rank T is 3 for DRUM and DegreEmbed, cause Neural LP is not developed using the low-rank approximation method.

5.2. Results

5.2.1. Results on KGC task

We compare our DegreEmbed to several baseline models on the KGC benchmark datasets as stated in the Section 5.1.2 and Section 5.1.3. Our results on the selected benchmark datasets are summarized in Table 3 and Table 4.

We notice that except that ComplEx [12] produces the best result among all methods on UMLS under the evaluation of Hit@1, all models are outperformed by DegreEmbed with a clear margin, especially on the dataset Kinship where we can see an about 10% improvement on some metrics. Besides, our model also achieves high performance on two real world KG dataset as shown in the appendix. As expected, incorporating entity embedding enhances the expressive power of DegreEmbed and thus benefits to reasoning on heterogeneous KGs.

Table 3
Knowledge graph completion performance comparison. Hit@k (H@k) is in %.

	Family				Kinship				UMLS			
	MRR	H@1	H@3	H@10	MRR	H@1	H@3	H@10	MRR	H@1	H@3	H@10
TransE	.34	7	53	86	.26	1	42	76	.57	28	84	96
DistMult	.58	39	71	91	.51	36	57	87	.73	63	81	90
ComplEx	.83	72	94	98	.61	44	71	92	.79	**69**	87	95
TuckER	.43	28	52	72	.60	46	70	86	.73	63	81	91
RotatE	.90	85	95	99	.65	50	76	93	.73	64	82	94
RNNLogic	.93	91	95	99	.64	50	73	93	.75	63	83	92
Neural LP	.91	86	95	99	.62	48	69	91	.75	62	86	92
DRUM	.94	90	98	99	.58	43	67	90	.80	66	94	97
DegreEmbed	**.95**	**91**	**99**	**100**	**.70**	**57**	**79**	**94**	**.80**	65	**94**	**98**

Table 4
Knowledge graph completion results on FB15K-237 and WN18. Hit@k is in %.

	FB15K-237				WN18			
	MRR	Hit@1	Hit@3	Hit@10	MRR	Hit@1	Hit@3	Hit@10
TransE	.15	5	19	25	.36	4	63	81
DistMult	.25	17	28	42	.71	56	83	93
ComplEx	.26	17	29	44	.90	88	92	94
TuckER	.36	27	39	36	.94	93	94	95
RotatE	.34	24	38	53	95	94	95	96
RNNLogic	.29	21	31	43	.94	93	94	96
Neural LP	.25	19	27	37	.94	93	94	95
DRUM	.25	19	28	38	.54	49	54	66
DegreEmbed	.25	19	27	38	.95	94	95	97

Notably, DegreEmbed not only is capable of producing state-of-the-art results on KGC task thanks to the degree embedding of entities, but also maintains the advantage of logic rule learning that enables our model to be interpretable to human, which is of vital significance in current research of intelligent systems. We will show the experiment results on the quality of mined rules by DegreEmbed later.

5.2.2. *Quality of mined rules*

Apart from reaching state-of-the-art performance on KGC task which is largely thanks to the mechanism of entity embedding, our DegreEmbed, as a knowledge graph reasoning model based on logic rule mining, is of excellent interpretability as well. Our work follows the Neural LP [16] framework, which successfully combines structure learning and parameter learning to generate rules along with confidence scores.

In this section, we report evaluation results on explanations of our model where some of the rules learned by DegreEmbed are shown. As for evaluation metric, we use the indicator *saturation* to objectively assess the quality of mined rules in a computable manner. We conduct two separate KGC experiments for generating the logic rules where the only difference is whether the inverted queries are learned. For better visualization purposes, experiments are done on the Family dataset, while other datasets such as UMLS produce similar results.

We sort the rules mined by DegreEmbed by their normalized confidence scores, which are computed by dividing by the maximum confidence of rules for each relation, and show top rules without augmented queries in Table 6 and rules with inverted relations in Table 7. Saturations of rules according to specific relations are shown in Table 5.

Table 5

Saturations of the Family dataset. $\gamma_q^{\mathbf{p}_l}$, $\delta_q^{\mathbf{p}_l}$, $\eta_q^{\mathbf{p}_l}$ are *macro*, *micro* and *comprehensive* saturations. The results relating to a specific relation are sorted by the comprehensive saturation in descending order.

Rule	$\Rightarrow$	Relation	$\gamma_q^{\mathbf{p}_l}$	$\delta_q^{\mathbf{p}_l}$	$\eta_q^{\mathbf{p}_l}$
$X \xRightarrow{\texttt{nephewOf}} Z \xRightarrow{\texttt{brotherOf}} Y$	$\Rightarrow$	$X \xRightarrow{\texttt{nephewOf}} Y$	.86	.25	.21
$X \xRightarrow{\texttt{nephewOf}} Z \xRightarrow{\texttt{sisterOf}} Y$	$\Rightarrow$		.79	.22	.17
$X \xRightarrow{\texttt{brotherOf}} Z \xRightarrow{\texttt{nephewOf}} Y$	$\Rightarrow$		.79	.21	.16
$X \xRightarrow{\texttt{brotherOf}} Z \xRightarrow{\texttt{nieceOf}} Y$	$\Rightarrow$		.72	.17	.12
$X \xRightarrow{\texttt{sonOf}} Z \xRightarrow{\texttt{brotherOf}} Y$	$\Rightarrow$		.64	.10	.06
$X \xRightarrow{\texttt{sonOf}} Z \xRightarrow{\texttt{sisterOf}} Y$	$\Rightarrow$		.36	.05	.02
$X \xRightarrow{\texttt{sisterOf}} Z \xRightarrow{\texttt{sonOf}} Y$	$\Rightarrow$	$X \xRightarrow{\texttt{daughterOf}} Y$	.68	.25	.17
$X \xRightarrow{\texttt{sisterOf}} Z \xRightarrow{\texttt{daughterOf}} Y$	$\Rightarrow$		.61	.20	.12
$X \xRightarrow{\texttt{daughterOf}} Z \xRightarrow{\texttt{husbandOf}} Y$	$\Rightarrow$		.46	.15	.07
$X \xRightarrow{\texttt{daughterOf}} Z \xRightarrow{\texttt{wifeOf}} Y$	$\Rightarrow$		.46	.14	.06
$X \xRightarrow{\texttt{brotherOf}} Z \xRightarrow{\texttt{brotherOf}} Y$	$\Rightarrow$	$X \xRightarrow{\texttt{brotherOf}} Y$	.86	.14	.12
$X \xRightarrow{\texttt{nephewOf}} Z \xRightarrow{\texttt{uncleOf}} Y$	$\Rightarrow$		.77	.13	.10
$X \xRightarrow{\texttt{brotherOf}} Z \xRightarrow{\texttt{sisterOf}} Y$	$\Rightarrow$		.81	.13	.10
$X \xRightarrow{\texttt{sonOf}} Z \xRightarrow{\texttt{fatherOf}} Y$	$\Rightarrow$		1.00	.08	.08
$X \xRightarrow{\texttt{nephewOf}} Z \xRightarrow{\texttt{auntOf}} Y$	$\Rightarrow$		.68	.11	.08
$X \xRightarrow{\texttt{sonOf}} Z \xRightarrow{\texttt{motherOf}} Y$	$\Rightarrow$		.98	.07	.07
$X \xRightarrow{\texttt{sisterOf}} Z \xRightarrow{\texttt{uncleOf}} Y$	$\Rightarrow$	$X \xRightarrow{\texttt{auntOf}} Y$	.89	.26	.23
$X \xRightarrow{\texttt{sisterOf}} Z \xRightarrow{\texttt{auntOf}} Y$	$\Rightarrow$		.85	.22	.19
$X \xRightarrow{\texttt{auntOf}} Z \xRightarrow{\texttt{brotherOf}} Y$	$\Rightarrow$		.83	.21	.17
$X \xRightarrow{\texttt{auntOf}} Z \xRightarrow{\texttt{sisterOf}} Y$	$\Rightarrow$		.75	.18	.13
$X \xRightarrow{\texttt{sisterOf}} Z \xRightarrow{\texttt{fatherOf}} Y$	$\Rightarrow$		.66	.09	.06
$X \xRightarrow{\texttt{sisterOf}} Z \xRightarrow{\texttt{motherOf}} Y$	$\Rightarrow$		.34	.05	.02
$X \xRightarrow{\texttt{sisterOf}} Z \xRightarrow{\texttt{brotherOf}} Y$	$\Rightarrow$	$X \xRightarrow{\texttt{sisterOf}} Y$	.89	.15	.13
$X \xRightarrow{\texttt{sisterOf}} Z \xRightarrow{\texttt{sisterOf}} Y$	$\Rightarrow$		.84	.14	.12
$X \xRightarrow{\texttt{nieceOf}} Z \xRightarrow{\texttt{uncleOf}} Y$	$\Rightarrow$		.78	.13	.10
$X \xRightarrow{\texttt{auntOf}} Z \xRightarrow{\texttt{nephewOf}} Y$	$\Rightarrow$		.67	.12	.08
$X \xRightarrow{\texttt{daughterOf}} Z \xRightarrow{\texttt{fatherOf}} Y$	$\Rightarrow$		1.00	.07	.07
$X \xRightarrow{\texttt{daughterOf}} Z \xRightarrow{\texttt{motherOf}} Y$	$\Rightarrow$		.99	.07	.07
$X \xRightarrow{\texttt{brotherOf}} Z \xRightarrow{\texttt{sonOf}} Y$	$\Rightarrow$	$X \xRightarrow{\texttt{sonOf}} Y$	.64	.24	.15
$X \xRightarrow{\texttt{brotherOf}} Z \xRightarrow{\texttt{daughterOf}} Y$	$\Rightarrow$		.56	.19	.10
$X \xRightarrow{\texttt{sonOf}} Z \xRightarrow{\texttt{husbandOf}} Y$	$\Rightarrow$		.46	.16	.08
$X \xRightarrow{\texttt{sonOf}} Z \xRightarrow{\texttt{wifeOf}} Y$	$\Rightarrow$		.46	.14	.06
$X \xRightarrow{\texttt{nephewOf}} Z \xRightarrow{\texttt{brotherOf}} Y$	$\Rightarrow$		.39	.12	.05

Table 6

Top rules without inverted queries mined by DegreEmbed on the Family dataset.

Rule	⇒	Relation	Confidence
$X \xrightarrow{\texttt{brotherOf}} Z \xrightarrow{\texttt{nephewOf}} Y$	⇒	$X \xrightarrow{\texttt{nephewOf}} Y$	1.00
$X \xrightarrow{\texttt{brotherOf}} Z \xrightarrow{\texttt{nieceOf}} Y$	⇒		0.88
$X \xrightarrow{\texttt{sonOf}} Z \xrightarrow{\texttt{sisterOf}} Y$	⇒		0.34
$X \xrightarrow{\texttt{sonOf}} Z \xrightarrow{\texttt{brotherOf}} Y$	⇒		0.16
$X \xrightarrow{\texttt{nephewOf}} Z \xrightarrow{\texttt{sisterOf}} Y$	⇒		0.13
$X \xrightarrow{\texttt{sisterOf}} Z \xrightarrow{\texttt{sonOf}} Y$	⇒	$X \xrightarrow{\texttt{daughterOf}} Y$	1.00
$X \xrightarrow{\texttt{sisterOf}} Z \xrightarrow{\texttt{daughterOf}} Y$	⇒		0.84
$X \xrightarrow{\texttt{daughterOf}} Z \xrightarrow{\texttt{wifeOf}} Y$	⇒		0.72
$X \xrightarrow{\texttt{daughterOf}} Z \xrightarrow{\texttt{husbandOf}} Y$	⇒		0.24
$X \xrightarrow{\texttt{brotherOf}} Z \xrightarrow{\texttt{sisterOf}} Y$	⇒	$X \xrightarrow{\texttt{brotherOf}} Y$	1.00
$X \xrightarrow{\texttt{brotherOf}} Z \xrightarrow{\texttt{brotherOf}} Y$	⇒		0.81
$X \xrightarrow{\texttt{sonOf}} Z \xrightarrow{\texttt{motherOf}} Y$	⇒		0.55
$X \xrightarrow{\texttt{sonOf}} Z \xrightarrow{\texttt{fatherOf}} Y$	⇒		0.18
$X \xrightarrow{\texttt{sisterOf}} Z \xrightarrow{\texttt{motherOf}} Y$	⇒	$X \xrightarrow{\texttt{auntOf}} Y$	1.00
$X \xrightarrow{\texttt{sisterOf}} Z \xrightarrow{\texttt{fatherOf}} Y$	⇒		0.77
$X \xrightarrow{\texttt{sisterOf}} Z \xrightarrow{\texttt{auntOf}} Y$	⇒		0.77
$X \xrightarrow{\texttt{sisterOf}} Z \xrightarrow{\texttt{uncleOf}} Y$	⇒		0.31
$X \xrightarrow{\texttt{sisterOf}} Z \xrightarrow{\texttt{sisterOf}} Y$	⇒	$X \xrightarrow{\texttt{sisterOf}} Y$	1.00
$X \xrightarrow{\texttt{sisterOf}} Z \xrightarrow{\texttt{brotherOf}} Y$	⇒		0.90
$X \xrightarrow{\texttt{sisterOf}} Z \xrightarrow{\texttt{motherOf}} Y$	⇒		0.39
$X \xrightarrow{\texttt{brotherOf}} Z \xrightarrow{\texttt{sonOf}} Y$	⇒	$X \xrightarrow{\texttt{sonOf}} Y$	1.00
$X \xrightarrow{\texttt{brotherOf}} Z \xrightarrow{\texttt{daughterOf}} Y$	⇒		0.67
$X \xrightarrow{\texttt{sonOf}} Z \xrightarrow{\texttt{husbandOf}} Y$	⇒		0.56
$X \xrightarrow{\texttt{sonOf}} Z \xrightarrow{\texttt{wifeOf}} Y$	⇒		0.39

By referring to the results given by computing saturations, we can see the rules mined by our model solidly agree with the ones with high saturation level, For example, the normalized confidences of the rule $\texttt{sisterOf}(X, Z) \wedge \texttt{sonOf}(Z, Y) \Rightarrow \texttt{daughterOf}(X, Y)$ and the rule $\texttt{sisterOf}(X, Z) \wedge \texttt{daughterOf}(Z, Y) \Rightarrow \texttt{daughterOf}(X, Y)$ are 1.0 and 0.84 respectively, whose saturations also rank the first and second places in Table 5. Meanwhile, our model obviously gets rid of the noises rendered by the heterogeneousness of the dataset through blending entity attributes (*e.g.* gender of entities) into rule learning, *e.g.*, the rules mined for predicting the relation `daughterOf`, such as `sisterOf` ∧ `sonOf` and `sisterOf` ∧ `daughterOf`, all show up with a female-type relation at the first hop. In this case, our DegreEmbed model is capable of learning meaningful rules, which indeed proves the interpretability of our model.

6. Conclusions

In this paper, a logic rule learning model called DegreEmbed has been proposed for reasoning more effectively in heterogeneous knowledge graphs, where there exist entities and relations of different types.

Table 7

Top rules with inverted queries mined by DegreEmbed on the Family dataset.

Rule	⇒	Relation	Confidence
$X \xrightarrow{\texttt{inv\_sisterOf}} Z \xrightarrow{\texttt{inv\_uncleOf}} Y$	$\Rightarrow$	$X \xrightarrow{\texttt{nephewOf}} Y$	1.00
$X \xrightarrow{\texttt{brotherOf}} Z \xrightarrow{\texttt{inv\_auntOf}} Y$	$\Rightarrow$		0.39
$X \xrightarrow{\texttt{inv\_sisterOf}} Z \xrightarrow{\texttt{inv\_auntOf}} Y$	$\Rightarrow$		0.36
$X \xrightarrow{\texttt{inv\_brotherOf}} Z \xrightarrow{\texttt{inv\_auntOf}} Y$	$\Rightarrow$		0.32
$X \xrightarrow{\texttt{inv\_sisterOf}} Z \xrightarrow{\texttt{nieceOf}} Y$	$\Rightarrow$		0.29
$X \xrightarrow{\texttt{brotherOf}} Z \xrightarrow{\texttt{inv\_uncleOf}} Y$	$\Rightarrow$		0.22
$X \xrightarrow{\texttt{inv\_sisterOf}} Z \xrightarrow{\texttt{inv\_motherOf}} Y$	$\Rightarrow$	$X \xrightarrow{\texttt{daughterOf}} Y$	1.00
$X \xrightarrow{\texttt{inv\_sisterOf}} Z \xrightarrow{\texttt{inv\_fatherOf}} Y$	$\Rightarrow$		0.67
$X \xrightarrow{\texttt{inv\_brotherOf}} Z \xrightarrow{\texttt{inv\_fatherOf}} Y$	$\Rightarrow$		0.31
$X \xrightarrow{\texttt{sisterOf}} Z \xrightarrow{\texttt{inv\_fatherOf}} Y$	$\Rightarrow$		0.26
$X \xrightarrow{\texttt{inv\_brotherOf}} Z \xrightarrow{\texttt{inv\_motherOf}} Y$	$\Rightarrow$		0.18
$X \xrightarrow{\texttt{sisterOf}} Z \xrightarrow{\texttt{inv\_motherOf}} Y$	$\Rightarrow$		0.17
$X \xrightarrow{\texttt{brotherOf}} Z \xrightarrow{\texttt{inv\_sisterOf}} Y$	$\Rightarrow$	$X \xrightarrow{\texttt{brotherOf}} Y$	1.00
$X \xrightarrow{\texttt{inv\_brotherOf}} Z \xrightarrow{\texttt{inv\_brotherOf}} Y$	$\Rightarrow$		0.57
$X \xrightarrow{\texttt{brotherOf}} Z \xrightarrow{\texttt{brotherOf}} Y$	$\Rightarrow$		0.55
$X \xrightarrow{\texttt{brotherOf}} Z \xrightarrow{\texttt{sisterOf}} Y$	$\Rightarrow$		0.37
$X \xrightarrow{\texttt{inv\_brotherOf}} Z \xrightarrow{\texttt{inv\_sisterOf}} Y$	$\Rightarrow$		0.20
$X \xrightarrow{\texttt{inv\_sisterOf}} Z \xrightarrow{\texttt{motherOf}} Y$	$\Rightarrow$	$X \xrightarrow{\texttt{auntOf}} Y$	1.00
$X \xrightarrow{\texttt{sisterOf}} Z \xrightarrow{\texttt{fatherOf}} Y$	$\Rightarrow$		0.26
$X \xrightarrow{\texttt{inv\_sisterOf}} Z \xrightarrow{\texttt{inv\_nephewOf}} Y$	$\Rightarrow$		0.26
$X \xrightarrow{\texttt{inv\_sisterOf}} Z \xrightarrow{\texttt{inv\_nieceOf}} Y$	$\Rightarrow$		0.23
$X \xrightarrow{\texttt{inv\_sisterOf}} Z \xrightarrow{\texttt{inv\_daughterOf}} Y$	$\Rightarrow$		0.18
$X \xrightarrow{\texttt{sisterOf}} Z \xrightarrow{\texttt{sisterOf}} Y$	$\Rightarrow$	$X \xrightarrow{\texttt{sisterOf}} Y$	1.00
$X \xrightarrow{\texttt{sisterOf}} Z \xrightarrow{\texttt{inv\_brotherOf}} Y$	$\Rightarrow$		0.72
$X \xrightarrow{\texttt{inv\_sisterOf}} Z \xrightarrow{\texttt{inv\_sisterOf}} Y$	$\Rightarrow$		0.51
$X \xrightarrow{\texttt{inv\_brotherOf}} Z \xrightarrow{\texttt{inv\_sisterOf}} Y$	$\Rightarrow$		0.16
$X \xrightarrow{\texttt{inv\_sisterOf}} Z \xrightarrow{\texttt{inv\_brotherOf}} Y$	$\Rightarrow$		0.10
$X \xrightarrow{\texttt{inv\_brotherOf}} Z \xrightarrow{\texttt{inv\_motherOf}} Y$	$\Rightarrow$	$X \xrightarrow{\texttt{sonOf}} Y$	1.00
$X \xrightarrow{\texttt{inv\_sisterOf}} Z \xrightarrow{\texttt{inv\_fatherOf}} Y$	$\Rightarrow$		0.43
$X \xrightarrow{\texttt{brotherOf}} Z \xrightarrow{\texttt{inv\_motherOf}} Y$	$\Rightarrow$		0.37
$X \xrightarrow{\texttt{inv\_sisterOf}} Z \xrightarrow{\texttt{inv\_motherOf}} Y$	$\Rightarrow$		0.31
$X \xrightarrow{\texttt{brotherOf}} Z \xrightarrow{\texttt{inv\_fatherOf}} Y$	$\Rightarrow$		0.28
$X \xrightarrow{\texttt{inv\_brotherOf}} Z \xrightarrow{\texttt{inv\_fatherOf}} Y$	$\Rightarrow$		0.27

Based on mining logic rules, DegreEmbed simultaneously leverages latent knowledge of entities by learning embedding vectors for them, where the degrees of the entities are closely observed. Experiment results show that our model benefits from the advantages of both embedding-based methods and rule learning systems, as one can see DegreEmbed outperforms the state-of-the-art models with a clear margin, and it produces high-quality rules with great interpretability. In the future, we would like to optimize the way of entity embedding to increase the expressive power of logic rule learning models for knowledge graph reasoning.

Acknowledgements

The work of this paper is supported by the "National Key R&D Program of China" (2020YFB2009502), "the Fundamental Research Funds for the Central Universities" (Grant No. HIT.NSRIF.2020098).

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